www.nature.com/scientificreports

scientific reports

OPEN

Visual outdoor space perception

J. Farley Norman✉, Maria Carmichael, Evan Hagan, Yulia Mishchuk, Emily N. Bryant, Alejandro B. Ramirez & Avery Sutton

Visual space perception has been a topic of sustained research since the nineteenth century. Much of this research into the geometry of visual space, however, required observers to make judgments about spatial relationships between isolated points in total darkness. While a sizeable number of previous investigations have now explored visual space perception in outdoor natural environments, nearly all of the previous investigations evaluating the curvature of visual space have utilized only small numbers of observers. In the current experiment, a large number (30) of observers adjusted triangular configurations of markers in an outdoor field until they appeared either as equilateral or right triangles in depth. There was a wide range of outcomes, such that the observers' judgments were consistent with elliptic, Euclidean, and hyperbolic geometry. There is thus no single consistent relationship between physical space and perceived space. Furthermore, the geometry of visual space frequently changes as the size of spatial configurations is varied—for many observers, judgments for small configurations are consistent with elliptic or Euclidean geometry while judgments for large configurations are frequently consistent with hyperbolic geometry.

In a series of articles, Albert Blank[1,2] described several methods involving triangular spatial configurations by which one can evaluate the geometry of visual space. Koenderink, van Doorn, and Lappin[3] have similarly used tasks involving triangles to determine which geometry is consistent with the perception of spatial relationships. Blank's methods have been used widely for the past 60 years[4–7]. In one experiment, Blank[2] showed observers three small "starlike" lights (referred to as A, B, and C; see Fig. 1). It is important to keep in mind that in Euclidean geometry, the length of the side AC equals exactly twice the length of the distance between the midpoints of the lengths AB and BC. Therefore, in Blank's experiment, the seven observers were first asked to adjust the positions of two lights (points D and E) so that they appeared to bisect the lengths AB and BC. The observers then were asked to position a new light (F) from A along the path AC at a distance from A that appeared to equal the distance between the midpoints DE. They then positioned an additional new light (G) from C along the path AC at a distance from C that similarly appeared to equal the distance between the midpoints DE. If an observer's perception of space is Euclidean then these two adjusted positions (F and G) should coincide (upper illustration in Fig. 1). If the perceived space is hyperbolic (lower left illustration in Fig. 1), then the points F and G do not overlap (i.e., F is closest to A and G is closest to C). If, however, points F and G overlap (F is closer to C, while G is closer to A, see lower right illustration in Fig. 1), then the observer's perceived space is elliptic (for a description of hyperbolic and elliptic surfaces, see Koenderink[8] and Hilbert and Cohn-Vossen[9]). In Blank's[2] experiment, six of the seven observers' judgments were consistent with hyperbolic geometry, while one observer's judgments were consistent with Euclidean geometry. It is important to keep in mind, however, that in this experiment, the seven observers were making judgments about spatial configurations of point lights in total darkness -- there is no reason to expect that this result would generalize to lighted situations in natural outdoor or indoor environments. Furthermore, only one size (of the triangular configuration) was judged -- Blank's single triangle was relatively small (approximately 0.6 × 2 m). The results of Koenderink et al.[3] have demonstrated that the geometry of visual space is probably affected by the size of the configuration judged (see Fig. 6 of Koenderink et al.).

In 1976, Battro et al.[4] attempted to conduct what would have been a very interesting experiment, a study of spatial relationships outdoors in lighted, everyday conditions -- see the description of this experiment in Sect. 3.2 of Battro et al. (pp. 20–21) entitled "Blank's visual triangles". To facilitate the interpretation of Battro et al.'s conclusions, let us refer to the length of DE as d (see our Fig. 1 and Fig. 8 of Battro et al.) and similarly refer to the length of AC as c. As discussed earlier, in Euclidean geometry, $2d = c$. In hyperbolic geometry, $2d < c$, and in elliptic geometry, $2d > c$ (see Blank[2]). Unfortunately, there are substantial ambiguities about both the method and results of Battro et al. that prevent meaningful conclusions. With regards to the method, Blank asked his observers to estimate the midpoints D and E and then transport that perceived distance (between D and E) onto the side AC to determine whether the curvature of visual space is positive (elliptic), negative (hyperbolic), or zero (Euclidean). In the experiment conducted by Battro et al., we don't know whether the experimenters marked the actual physical midpoints D and E with stakes or whether the midpoints were perceptually determined

Department of Psychological Sciences, Western Kentucky University, Bowling Green, KY 42101-2030, USA. ✉email: farley.norman@wku.edu

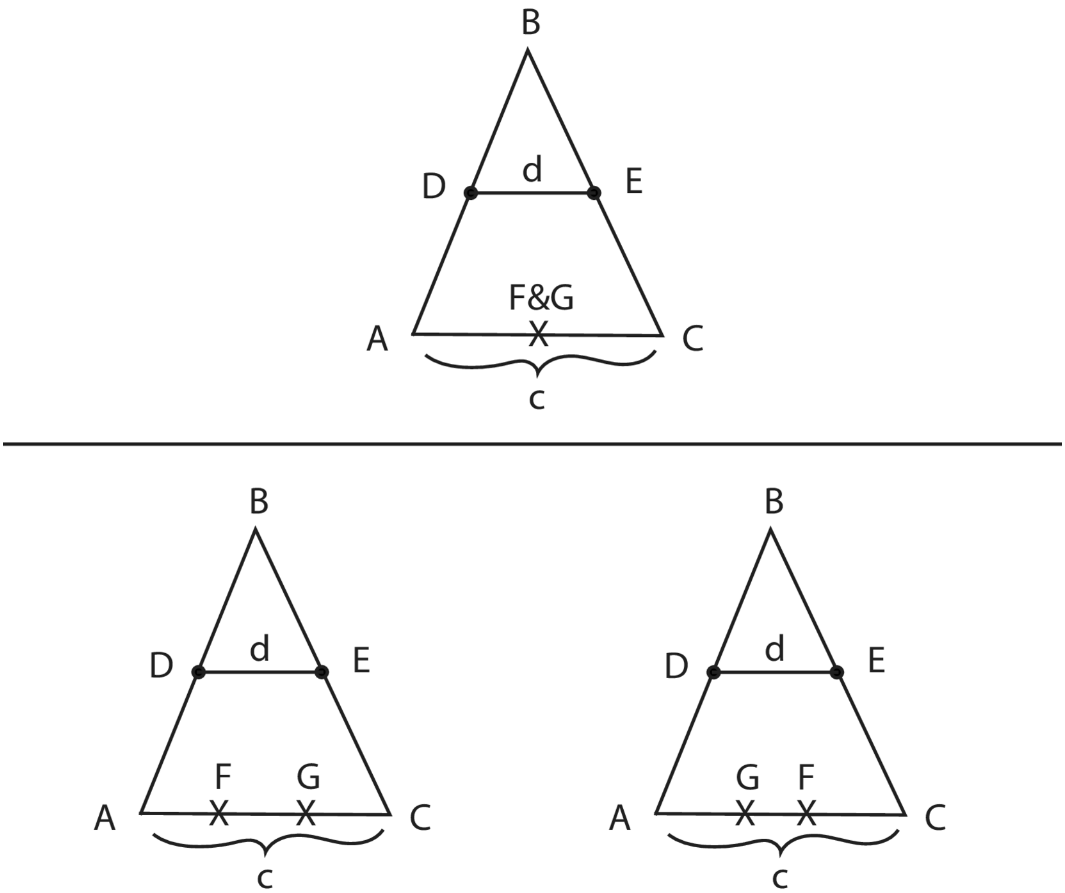

Fig. 1. Illustrations of possible outcomes of the task described by Blank[2]. The observer views a triangular configuration defined by points A, B, and C. The observer first estimates the midpoints of sides AB and BC. These midpoints are referred to by points D and E. The distance d indicates the distance between the perceived midpoints. The observer then marks a point F that is located at a perceived distance d from A along the path from A to C. The observer then marks a point G that is located at a perceived distance d from C along the path from C to A. If the observers' judgments are consistent with Euclidean geometry (top), then the adjusted points F and G should coincide, both being located at a point halfway between A and C. If the observers' judgments are consistent with hyperbolic geometry (lower left), then the adjusted points F and G will not coincide and F will be located closer to A, while G will be located closer to C. If the observers' judgments are consistent with elliptic geometry (lower right), then the adjusted points F and G will not coincide and G will be located closer to A, while F will be located closer to C.

for each observer (as Blank did). Battro et al. said simply (p. 21) "stakes D and E bisected the sides a and b" -- was the bisection physical or perceptual? We simply don't know today. Furthermore, when describing the task performed by the observers, Battro et al. (p. 21) said "As usual the subject was told to tell the experimenter to fix the stake at a distance from A that equaled the distance d. Six measures were taken, three when the experimenter walked from A to E, and three when he walked from E to A". This makes no sense, and is an obvious error of some kind (whether in writing or in implementation of the method). To correctly implement the method of Blank[2], the experimenter should have walked from "A to C" or from "C to A". In short, we do not really know whether the task described by Blank for determining the curvature of visual space was implemented correctly by Battro et al. The conclusions of Battro et al. are shown in a table entitled "Frequency of Euclidean, spheric, and hyperbolic visual triangles in open space" (see Table 9 of Battro et al.). Battro et al. concluded that there were significant differences in the curvature of perceived space -- for example, those authors concluded that of ten observers who judged triangles that were 15 m wide on side AC (what were the lengths of sides AB and BC?), five observers' judgments were consistent with hyperbolic geometry, five observers' judgments were consistent with elliptic geometry, and none of the observers' judgments were consistent with Euclidean geometry. These conclusions are certainly interesting and would apparently document important differences in the visual perception of space. Unfortunately, however, no actual data are presented in Battro et al.'s description of this experiment. There are no figures, plots, or tables of numerical judgments. In contrast, one can easily see numerical descriptions of the observers' judgments in Blank's experiment (e.g., see Table 1 of Blank[2]). Battro et al. concluded that five observers' judgments were consistent with elliptic geometry, that is that $2d > c$, for 15 m triangles (where d supposedly represented the observers' judgment). Was 2d a lot bigger than c (reflecting strong curvature of

visual space)? Was 2d meaningfully bigger than c? Was 2d only a little bigger than c (a small value, but perhaps reflecting a statistically significant deviation from Euclidean geometry)? There is no way to know now, since no actual numerical data was ever presented in the description of Battro et al.'s experiment. How far did 2d have to deviate from c in order for those authors to conclude that a particular observer's judgments were consistent with a particular type of geometry (i.e., what was the tolerance)?

The purpose of the current experiment was quite straightforward -- for the first time ever, to conduct an outdoor experiment (natural viewing conditions) with a large sample size, to determine the curvature of visual space using Blank's methods. In the current experiment, we used two of Blank's other methods[1] (also see Norman et al.[5]), which require observers to adjust distances within triangular spatial configurations until a particular triangle appears to be either an equilateral or right triangle. Many previous studies have only evaluated visual space by having observers look at points of light in totally dark rooms[2,6,7]. In the outdoor experiments that have been conducted and which report quantitative data[3,5], only small numbers of observers participated (3 observers in the study by Koenderink et al.[3] and 6 observers in the study by Norman et al.[5]). In an effort to better understand which geometries best characterize visual space perception, the current experiment used Blank's methods (in a natural outdoor environment) to evaluate the curvature of visual space for a large sample of 30 individual observers. In the process of performing Blank's[1] tasks to determine the curvature of space, observers are necessarily required to adjust distances between markers in order to create equilateral or isosceles right triangles. Many previous studies[5,10–14] have established that observers' judgments of distance are reliable and consistent over time. These reliabilities (standard deviation of repeated judgments divided by their mean) obtained for lighted viewing conditions range from 3.45 to 8.2% (average reliability across these six studies was 5.56%). Given that human observers' judgments of distance have been repeatedly shown to be reliable, we chose to manipulate the size of the experimental triangular configurations in the current experiment (rather than conduct repeated trials for a single size), because Koenderink et al.[3] have shown that size is likely to alter the curvature of visual space (and the purpose of the current experiment was to evaluate the curvature of visual space for a large sample of observers). Because of the large variability that exists in human distance perception[13,15–19], it is quite likely that such variability will extend to the curvature of visual space (since the tasks used in evaluating space perception involve the perception and matching of environmental distances).

Method
Apparatus and experimental stimuli
The outdoor triangular stimulus configurations were defined by the placement (relative to the observer) of two PVC (polyvinyl chloride) poles that were 1.56 m tall and 2.7 cm in diameter. These poles have been used in multiple previous investigations[5,10]. An Apple M2 MacBook Air was used to determine the random order of stimulus sizes for a given task and to record the observers' judgments after each trial. The experiment took place within an 89 m × 24 m grassy field (see Fig. 2) on the Western Kentucky University campus. This field was situated in between a campus building (Ogden College Hall) and East 14th Avenue.

Procedure
The observers participated in two tasks described by Blank[1] (see p. 914) to evaluate the curvature of visual space: (1) the equilateral triangle task, and (2) the isosceles right triangle task. Both tasks were subsequently used by Higashiyama[6] (see his Experiment 1), who asked his observers to perform these tasks while viewing point lights (light emitting diodes) in a totally dark room. In the equilateral task, the observer forms one vertex of the triangle. A single pole was placed to the left of the observers' line of sight at a fixed distance from the observer (at a distance of either 2.0, 6.5, 11.0, or 15.5 m). This range of distances was used previously by Norman et al.[5] and Koenderink et al.[3]. The observers' task was to indicate to an experimenter where to place a second pole, so that the resulting triangular configuration (observer, fixed pole on the left, and moveable pole on the right) appeared equilateral (so that all three sides of the resulting triangle in depth appeared identical in length). In the isosceles right triangle task, the observer's position again formed one vertex of a triangle. Again, a fixed pole was placed on the observer's left at a distance of 2.0, 6.5, 11.0, or 15.5 m. The observers' task in this case was to indicate to an experimenter where to place the second (moveable) pole on the right so that (1) the distance between the two poles appeared identical to the distance between the observer and the fixed pole, and (2) that the angle at the left pole (left vertex of the resulting triangle) appeared to be 90 degrees (i.e., a right angle). Before beginning the experimental tasks observers were both (1) given extensive verbal descriptions of the tasks and (2) shown examples of equilateral and isosceles right triangles on paper; these instructions were given by the first author (i.e., J. Farley Norman). The experiment did not begin until the first author was convinced that each observer thoroughly understood the tasks. At the beginning of each trial the observer was required to turn in the opposite direction and face away from the stimulus configuration so that they could not see the initial placement of the moveable pole. Then the moveable pole was placed at a random location within a square area 50% larger than the size of the triangle (e.g., for a triangle size of 6.5 m, the initial starting location of the moveable pole was randomly placed within an approximately 10 m x 10 m area, subject only to the restriction that the moveable pole be initially placed to the right of the fixed pole (remember from the earlier description of the task that the moveable pole was always located to the right of the fixed pole). Once the moveable pole was in position, the observer was told to turn around and face the initial stimulus configuration. During both tasks, the observers could take as much time as they wanted in order to make the stimulus triangles appear either equilateral or as isosceles right triangles in depth – a single trial frequently took 6 to 8 min as the observers first made initial (often large) adjustments to the location of the movable pole, then made a series of increasingly fine adjustments to the moveable pole's location until the adjusted stimulus configuration appeared to be exactly an equilateral or isosceles right triangle. The main dependent measure was the angle at the observer's vertex after the observer had created either a perceived equilateral or perceived isosceles right triangle (see Blank[1]). After each trial, the

Fig. 2. A photograph of the 89 m × 24 m grassy field where the experiment was conducted. This photograph was taken by the first author (J.F.N.) using an Apple iPhone XR digital camera.

experimenters measured the distances/lengths of all sides of the constructed triangle and used the law of cosines to calculate the angle at the observer's vertex. Accurate performance consistent with Euclidean geometry would be indicated by observer vertex angles of 60 and 45 degrees for the equilateral and isosceles right triangle tasks, respectively.

One trial was conducted for each of the four triangle sizes (2.0, 6.5, 11.0, and 15.5 m) for each of the two tasks (create what appeared to be equilateral and isosceles right triangles). The order of four triangle sizes was randomized for each task and the order of the two tasks (equilateral versus isosceles right triangle) was counterbalanced across individual observers. The location and viewing direction of the observer was also randomly varied across trials, such that every trial was unique. All observers were told to simply create what they perceived to be equilateral or isosceles right triangles in depth, but this randomization of location and viewing direction nevertheless prevented the possibility of the observers' potential use of consistent landmarks across trials. The observers used binocular viewing when performing the two tasks. The entire experiment generally took between 45 min and one hour per observer. No feedback about performance was provided to the observers until after they had completed the experiment.

Observers

The observers were 30 adults from the local community. Two of the observers were undergraduate student coauthors; the remaining 28 observers were completely naïve with regards to the purposes and details of the experiment and knew nothing other than the fact that they were being asked to make judgments about outdoor distances. The observers had excellent visual acuity: the acuity of the observers (measured with a Precision Vision ETDRS 2195 eyechart) was −0.1 LogMAR (log minimum angle of resolution; zero LogMAR represents normal visual acuity, while positive and negative values represent worse than and better than normal acuity, respectively). The study was approved by the Institutional Review Board of Western Kentucky University, and each observer signed an informed consent document prior to testing. Our research was carried out in accordance with the Code of Ethics of the World Medical Association (Declaration of Helsinki).

Results

Before considering the plots of the observers' results, it is very important to remember what the measured vertex angles at the observers' location actually indicate. Consider Fig. 3. Notice on the left that an equilateral triangle on a hyperbolic surface (curved like a horse saddle) has vertex angles less than 60 degrees, while equilateral triangles on elliptic surfaces (see right) possess vertex angles greater than 60 degrees. In Euclidean geometry (where space is not curved) equilateral triangles always have vertex angles of 60 degrees. Significant deviations away from 60 degrees for the equilateral triangle task (and analogous deviations away from 45 degrees for the

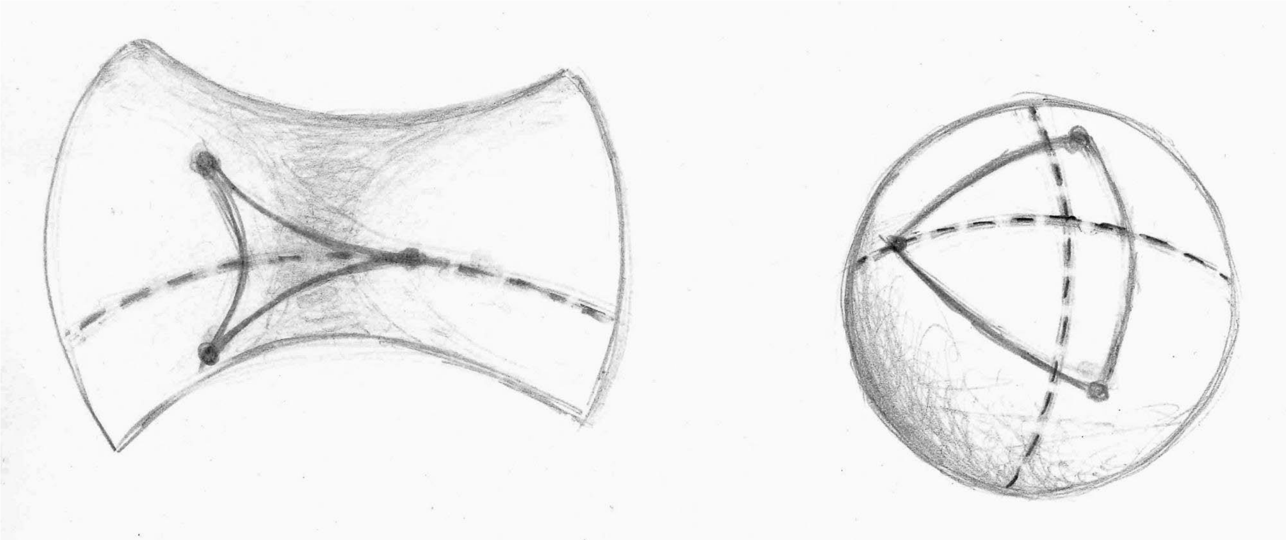

Fig. 3. Depictions of equilateral triangles on curved surfaces. Notice on the left that an equilateral triangle on a hyperbolic surface (curved like a horse saddle) has vertex angles less than 60 degrees, while equilateral triangles on elliptic surfaces (see right) possess vertex angles greater than 60 degrees. In Euclidean geometry (where space is not curved) equilateral triangles always have vertex angles of 60 degrees. The degree of deviation from 60 degrees indicates the magnitude of the curvature of space. This original drawing was made by the second author Maria Carmichael.

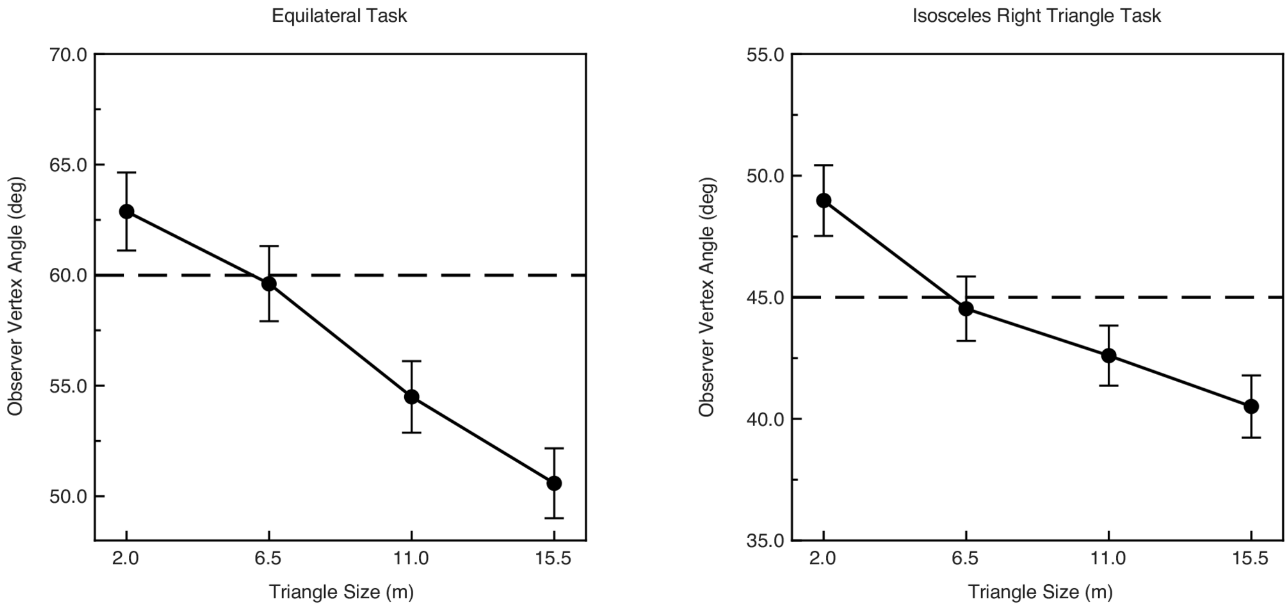

Fig. 4. Overall (mean) results for the equilateral triangle task (left) and isosceles right triangle task (right); see text and Blank[1]. The vertex angles at the observers' location are plotted as a function of the triangle size in meters. If the judgments are consistent with Euclidean geometry, the angles at the observers' location are 60 and 45 degrees for the equilateral and right triangle tasks, respectively. The dashed lines in each plot indicate the observer vertex angles consistent with Euclidean geometry. The error bars indicate ± 1 SE.

isosceles right triangle task) indicate that space is curved. Various aspects of the observers' results are shown in Figs. 4, 5, 6 and 7. The overall (i.e., mean) results for all 30 observers are shown in Fig. 4 for the equilateral task (left) and the right triangle task (right). The dashed lines indicate the observer vertex angles consistent with Euclidean geometry for each task (60 and 45 degrees for the equilateral and right triangle tasks, respectively). One can readily see that the observer vertex angles (after adjusting each stimulus triangle so that it appears to be either an equilateral or a right triangle in depth, see method section and Blank[1]) are high for small stimulus triangles and decrease in an almost linear fashion as the triangle size increases. This effect of stimulus size is

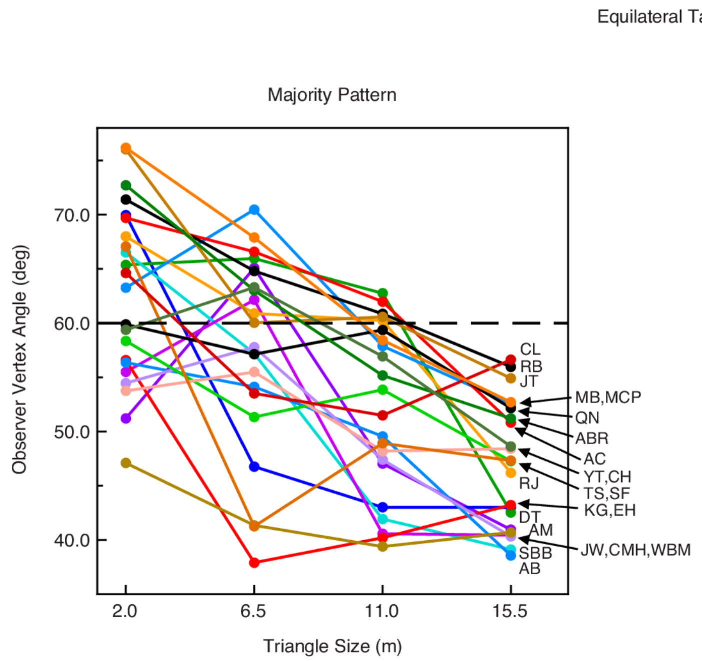

Fig. 5. Results for all 30 individual observers for the equilateral triangle task. The vertex angles at the observers' location are plotted as a function of the triangle size in meters. If the judgments are consistent with Euclidean geometry, the angles at the observers' location are 60 degrees. The dashed lines indicate the observer vertex angles consistent with Euclidean geometry. The observers who exhibit the typical pattern (highest vertex angles at the smaller stimulus sizes and whose judgments are consistent with hyperbolic geometry for the largest stimulus size) are shown on the left, while the remaining observers' (who exhibit other response patterns) judgments are shown on the right.

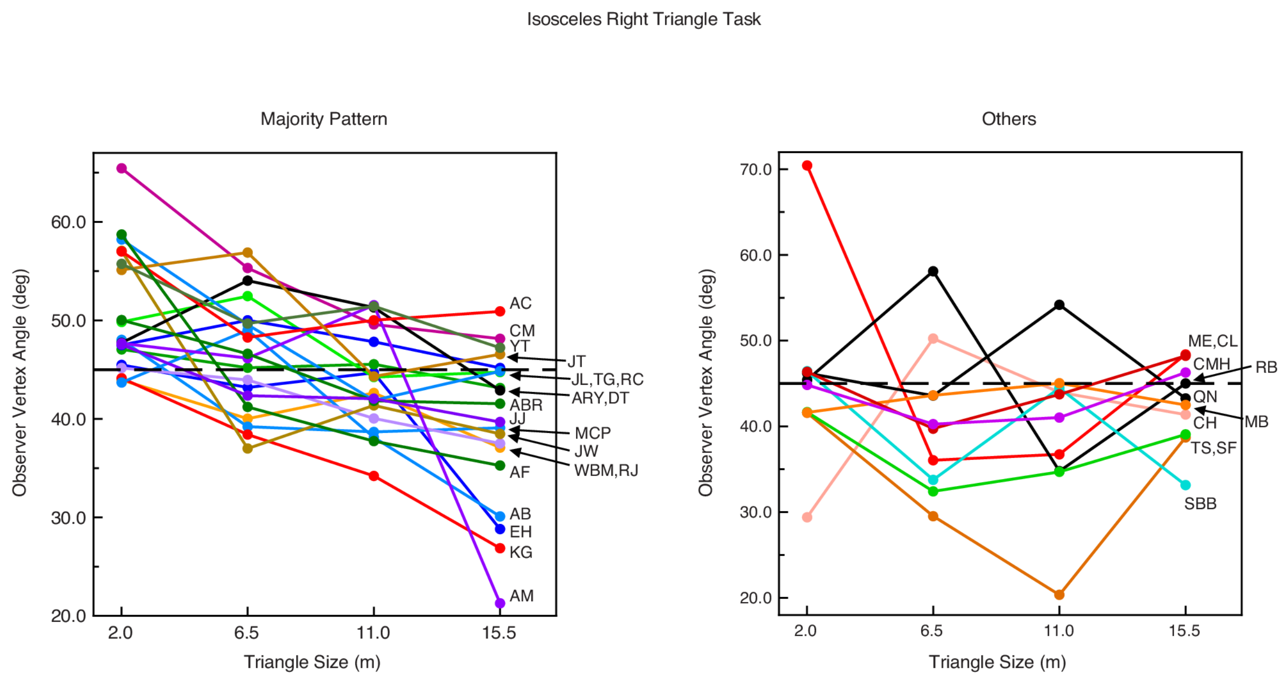

Fig. 6. Results for all 30 individual observers for the isosceles right triangle task. The vertex angles at the observers' location are plotted as a function of the triangle size in meters. If the judgments are consistent with Euclidean geometry, the angles at the observers' location are 45 degrees. The dashed lines indicate the observer vertex angles consistent with Euclidean geometry. The observers who exhibit the typical pattern (highest vertex angles at the smaller stimulus sizes and smaller vertex angles for the larger stimulus sizes) are shown on the left, while the remaining observers' (who exhibit other response patterns) judgments are shown on the right.

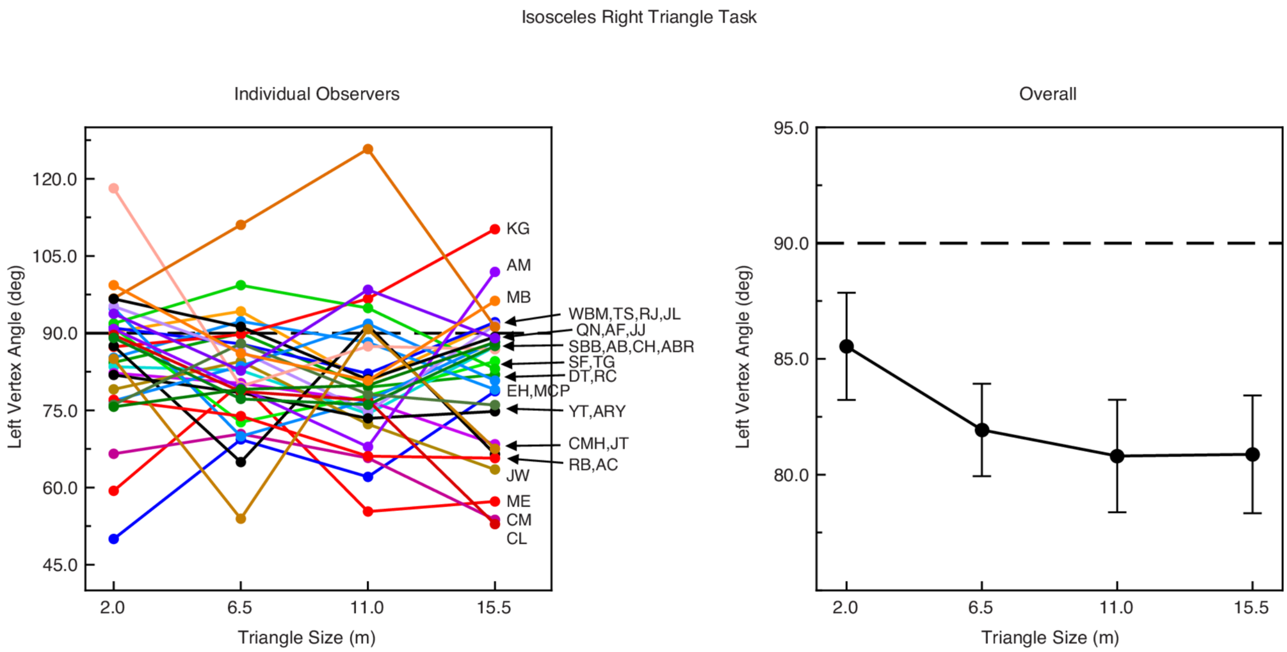

Fig. 7. Individual results (left) and overall results (right) for the isosceles right triangle task. The angles at the left triangle vertex are plotted as a function of the triangle size in meters. In this task the observers were asked to create triangles so that (1) the left and opposite sides of the triangle appeared equal in length, and (2) the left triangle vertex appeared to be a right angle (i.e., 90 degrees). One can readily see that while some individual observers created left vertex angles close to or larger than 90 degrees, that most of the observers produced left vertex angles smaller than 90 degrees (e.g., see overall results on the right). The error bars included in the overall results indicate ± 1 SE.

significant according to a two-way within-subjects analysis of variance (ANOVA; $F(3,87) = 29.3$, $p < .000001$, $\eta^2_p = 0.50$). The effect of task (equilateral or right triangle) upon the measured angles at the observers' location was expected and was significant also ($F(1,29) = 168.5$, $p < .000001$, $\eta^2_p = 0.85$). The task x triangle size interaction was not significant ($F(3,87) = 1.8$, $p = .15$, $\eta^2_p = 0.06$), indicating that the effect of stimulus size (2, 6.5, 11, 15.5 m) was similar for both tasks.

It is clear from Fig. 4 that the variations in triangle size produce relatively large changes in the magnitude of the angle measured at the observers' location. However, it is yet unclear whether these deviations from 60 and 45 degrees (for the equilateral and right triangle task, respectively) are sufficiently large to conclude that the observers' judgments are consistent with either elliptic or hyperbolic geometry. Given the previous results of Koenderink et al. and Norman et al.[5] with small sample sizes, we expected that small stimulus sizes would likely produce judgments consistent with elliptic geometry and that large stimulus sizes would likely produce judgments consistent with hyperbolic geometry (e.g., see Fig. 6 of Koenderink et al.). Because of these prior relevant results, we planned a number of a priori comparisons in advance of the study. We planned four comparisons (four one-sample t-tests, with the Bonferroni correction). First, we conducted one-sample t-tests for the largest stimulus size (15.5 m) for each of the two tasks, expecting that the observers' judgments would be consistent with hyperbolic geometry (i.e., observer vertex angles significantly lower than 60 and 45 degrees for the equilateral and right triangle tasks, respectively). Indeed, the one-sample t-tests for both tasks were significant (equilateral task: $t(29) = 5.95$, $p(1\text{-tailed}) = 0.000001$; right triangle: $t(29) = 3.51$, $p(1\text{-tailed}) = 0.0007$). We also conducted two final one-sample t-tests to determine whether the observers' judgments were consistent with elliptic geometry for the smallest (2 m) stimulus size (i.e., observer vertex angles significantly higher than 60 and 45 degrees for the equilateral and right triangle tasks, respectively). For the right triangle task, the observers' judgments were indeed consistent with elliptic geometry ($t(29) = 2.73$, $p(1\text{-tailed}) = 0.005$). This was not the case, however, for the equilateral triangle task; in this condition, the observers' vertex angles did not sufficiently deviate from 60 degrees ($t(29) = 1.63$, $p(1\text{-tailed}) = 0.057$) and thus, the observers' judgments in this condition (2 m, equilateral task) remained consistent with Euclidean geometry.

It is readily apparent from an inspection of Fig. 4, the ANOVA results, and the results of the one-sample t-tests that there are reliable similarities across observers in how they visually perceive space and spatial relationships: the observers' judgments, as a group, are consistent with hyperbolic geometry at the largest triangle size for both tasks and are consistent with elliptic geometry at the smallest triangle size for the right triangle task. Despite this overall similarity in judgment across observers, there were also large amounts of interobserver variability. This is illustrated in Fig. 5, which plots individual results (vertex angles at the observers' location) for the equilateral triangle task for all 30 observers. The pattern exhibited by the majority of the observers (22 out of 30) is shown on the left in Fig. 5, while other patterns (8 observers) are shown on the right. For the majority of observers

(left), the observer vertex angles are generally highest for the smaller stimulus triangles (2–6.5 m) and these observers' judgments are consistent with hyperbolic geometry (either mildly, moderately, or strongly) for the largest stimulus triangle (15.5 m). Other patterns (e.g., observer vertex angles increase as stimulus size increases; observers' judgments are generally not consistent with hyperbolic geometry at the largest stimulus size) are shown in Fig. 5 on the right. The analogous individual results of all 30 individual observers for the right triangle task are shown in Fig. 6. What is especially noticeable from an inspection of Figs. 5 and 6, however, are very large amounts of interobserver variability. For example, consider results for observers CL, RJ, and AB in Fig. 5. While all three of these observers' judgments are consistent with hyperbolic geometry at the largest stimulus size, it is obvious that CL's judgments are only mildly hyperbolic, RJ's are moderately hyperbolic, while AB's judgments are strongly hyperbolic -- the magnitudes of their deviation from Euclidean geometry are very different (deviations of 3.4, 13.8, and 21.4 degrees, respectively).

There is one final dependent measure of interest. In the right triangle task, the observers were instructed to create a triangle where (1) the left and opposite sides had the same perceived length and (2) that the angle at the left vertex appeared to be 90 degrees (i.e., a right angle). To what extent are human observers able to perceive, and thus create, right angles in outdoor environments? Since we measured the entire shape of each created triangle on every trial, we could easily evaluate the magnitude of the angles at the left vertex that appeared to be right angles to the observers. The observers' individual and overall left vertex angles are shown in Fig. 7. A one-way within-subjects ANOVA was conducted upon the individual results shown in Fig. 7. There was no effect of stimulus size upon the produced left vertex angles ($F(3,87) = 1.6$, $p = .19$; $\eta^2_p = 0.05$). A one-sample t-test revealed, however, that the observers' produced left vertex angles were significantly less than 90 degrees (see the overall results in the right panel of Fig. 7) at both extremes of stimulus size, 2m and 15.5 m (2m: $t(29) = 2.73$, p(2-tailed) = 0.011; 15.5 m: $t(29) = 3.51$, p(2-tailed) = 0.0015; once again Bonferroni-corrected critical p-values were used). It is interesting that while some individual observers were able to create accurate right angles in the right triangle task for some stimulus sizes (e.g., observer CL for the 2m stimulus size, observer QN at the 15.5 m size), that no observer was able to produce accurate right angles for all four stimulus sizes. It therefore appears that perceived right angles do not correspond to actual (i.e., physical) right angles even in a natural outdoor environment (a grassy field) with plenty of optical sources of information about depth and depth relationships.

Discussion

Given the substantial variability that occurs across observers in visual distance perception (e.g., Da Silva[16]; Da Silva, Ruiz, & Marques[17]; Norman et al.[15]; Norman, Adkins, Norman, Cox, & Rogers[18]; Norman, Dukes, Shapiro, & Peterson[19]; Table 1 of Lappin, Shelton, & Rieser[13]), it is probably not surprising that there were a wide variety of spatial configurations in the current experiment that were perceived to be equilateral or right triangles (since creating equilateral and isosceles right triangles requires observers to perceive and match environmental distances). Tasks involving triangular spatial configurations in depth are important, because they can inform us about the intrinsic nature of visual space[1,3]. The triangle experiment of Battro et al.[4] (pp. 20–21) was especially interesting, because those authors concluded (for example, for 15 m sized triangles) that while half of their observers' judgments were consistent with elliptic geometry, the remaining half of their observers' judgments were consistent with hyperbolic geometry (none of their observers' judgments in that condition were consistent with Euclidean geometry) even in full-cue outdoor environments. Unfortunately, as reviewed in the current introduction, the triangle experiment conducted by Battro et al. is ultimately unsatisfying, because the actual data upon which the authors categorized observers as being "hyperbolic" or "elliptic" was never described or published. Table 9 of Battro et al. is simply a description of the authors' conclusions -- no data tables, data descriptions, or data plots for that triangle experiment were included in the 1976 article to justify the conclusions. How strongly hyperbolic were the "hyperbolic" observers? How strongly elliptic were the "elliptic" observers? Because of such ambiguities, the overall question still remains unanswered. Do significant variations in the curvature of visually perceived space exist? Is the visual space of ordinary human observers hyperbolic for some observers and elliptic for others, even for the same task performed in the same environmental context? Does the magnitude of the intrinsic curvature of visual space vary dramatically across observers? The answers to such questions do not exist at present. Prior studies investigating the curvature of visual space outdoors, in full-cue environments, only used small sample sizes of three observers[3] or six observers[5]. Even if Battro et al. had plotted or described the actual data upon which their conclusions were based, those authors only evaluated the curvature of visual space for ten observers for particular stimulus conditions, whereas the current study evaluated the curvature of visual space for many more individual observers (30).

In their triangle experiment, Battro et al.[4] concluded (see their Table 9, p. 21) that the visual space of most (55%) of their observers was hyperbolic, while the visual space for almost all of the remaining observers (a further 42.5%) was elliptic (only one observer's visual space was labeled as Euclidean). Such a discrete classification of people into "hyperbolic", "elliptic", and "Euclidean" categories is overly simplistic. One can readily see from an examination of the individual results in our study (i.e., Figs. 5 and 6) that there is a continuous range of outcomes ranging from strongly elliptic, to moderately elliptic, to mildly elliptic, to Euclidean, to mildly hyperbolic, to moderately hyperbolic, and finally, to strongly hyperbolic. And even for single observers, the visual space of single individual observers frequently changes from elliptic to Euclidean to hyperbolic depending upon the stimulus size (also see Koenderink et al.[3]). One cannot simply categorize a person as being "hyperbolic", "elliptic", or Euclidean. Our results demonstrate that when the curvature of visual space is evaluated for a large sample of observers that there is a continuous distribution of outcomes—the visual space of observers does vary, but the changes in the curvature of visual space across observers are often quantitative, not simply qualitative as Battro et al.'s Table 9 suggests.

It is important to note that while there are large variations (Figs. 5 and 6) in the curvature of visual space across observers, that important similarities also occur. For example, the observers' vertex angles (which indicate

departures from Euclidean geometry) systematically decreased 12.3 degrees as the stimulus triangle sizes increased from 2 m to 15.5 m for the equilateral triangle task; there was an analogous corresponding overall decrease in observer vertex angles for the isosceles right triangle task (see Fig. 4 for the overall and systematic effect of stimulus size). The effect size (partial eta squared value) for this effect of stimulus size was a high 0.50. Fully 90 and 86.7% of the observers exhibited decreased observer vertex angles for the 15.5 m triangles (as compared to the analogous observer vertex angles produced for the 2 m triangles) for the equilateral and isosceles right triangle tasks, respectively.

If one compares the individual observers' judgments for the equilateral and isosceles right triangle tasks (i.e., compares Figs. 5 and 6), many of the observers (but not all) exhibit similar patterns of responses for both tasks. For example, out of the 24 observers whose judgments were consistent with hyperbolic geometry for the equilateral task for a stimulus size of 15.5 m, 17 of those observers' (71%) judgments were also consistent with hyperbolic geometry for the right triangle task for a stimulus size of 15.5 m. Also, out of the 16 observers whose judgments were consistent with elliptic geometry for the right triangle task at a stimulus size of 2 m, 11 of those observers' (69%) judgments were also consistent with elliptic geometry for the equilateral task at a stimulus size of 2 m. Nevertheless, it is true that a minority of the observers' visual space had different curvatures for the two tasks at a given size. A good example of such an observer would be observer AC. For a stimulus size of 15.5 m, this observer's (AC) judgments were consistent with hyperbolic geometry for the equilateral task, but were consistent with elliptic geometry for the right triangle task. This is indeed interesting. Even if this only occurs for a minority of observers (about 30% of the observers in the current experiment), what does it mean if someone's geometry of visual space depends upon the task that they perform? Jan Koenderink has proposed an explanation, which he calls the multiple visual worlds' hypothesis[20]. In this article Jan points out the essential ambiguity of perception, that our visual input (retinal images or optic array, if you like) is consistent with literally an infinite number of possible physical scenes. No single one of those infinite possible worlds is a more valid choice than another -- any of those infinite possibilities is just as likely as another. Jan proposes the possibility that in ordinary life perception is "like a *bundle* of possible visual worlds" (p. 3), and that we do not collapse the bundle or select a single one of these possibilities until we are forced to make a decision (e.g., when an experimenter requires you to make a unique response in a psychophysical experiment). Jan says (p. 3) "I think that in a great many cases perceptions are more of the multiple-visual-worlds variety than like the single guess… you don't notice the essential ambiguity of perception in real life". Jan concludes by referring to the type of task-dependent outcome that a minority of our observers demonstrated in the current experiment (e.g., observer AC, as referred to earlier in this paragraph); he says (p. 3) "That the multiple-visual-worlds option is indeed likely is suggested by the fact that a change of psychophysical method or task often leads to distinctly different results. This is not to say that observers actually entertain multiple-visual-worlds interpretations explicitly, but merely that they don't necessarily resolve ambiguities when this is not specifically required for some action or decision".

Conclusion

Human observers' perceptions of spatial relationships are rarely consistent with Euclidean geometry even in full-cue outdoor environments. In almost all instances, visual space is curved in either an elliptic or hyperbolic manner. The curvature of visual space itself is affected by size – judgments of small spatial configurations (e.g., 2 m) are often consistent with elliptic geometry, while judgments of larger spatial configurations (e.g., 15.5 m) are typically consistent with hyperbolic geometry.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request. Results for all individual participants are provided in the figures that accompany this article.

Received: 19 November 2024; Accepted: 28 March 2025
Published online: 04 April 2025

References

1. Blank, A. A. Analysis of experiments in binocular space perception. *J. Opt. Soc. Am.* **48**, 911–925 (1958).
2. Blank, A. A. Curvature of binocular visual space. An experiment. *J. Opt. Soc. Am.* **51**, 335–339 (1961).
3. Koenderink, J. J., van Doorn, A. J. & Lappin, J. S. Direct measurement of the curvature of visual space. *Perception* **29**, 69–79 (2000).
4. Battro, A. M., Netto, S. P. & Rozestraten, R. J. A. Riemannian geometries of variable curvature in visual space: Visual alleys, horopters, and triangles in big open fields. *Perception* **5**, 9–23 (1976).
5. Norman, J. F., Crabtree, C. E., Clayton, A. M. & Norman, H. F. The perception of distances and spatial relationships in natural outdoor environments. *Perception* **34**, 1315–1324 (2005).
6. Higashiyama, A. Variation of curvature in binocular visual space estimated by the triangle method. *Vis. Res.* **21**, 925–933 (1981).
7. Higashiyama, A. Curvature of binocular visual space: A modified method of right triangle. *Vis. Res.* **24**, 1713–1718 (1984).
8. Koenderink, J. J. *Solid Shape* (MIT Press, 1990).
9. Hilbert, D. & Cohn-Vossen, S. *Geometry and the Imagination* (Chelsea, 1983).
10. Dukes, J. M., Norman, J. F. & Shartzer, C. D. Visual distance perception indoors, outdoors, and in the dark. *Vis. Res.* **194**, 107992 (2022).
11. Norman, J. F., Dowell, C. J., Higginbotham, A. J., Fedorka, N. W. & Norman, H. F. Sex and age modulate the visual perception of distance. *Atten. Percept. Psychophys.* **80**, 2022–2032 (2018).
12. Norman, J. F. et al. The visual perception of exocentric distance in outdoor settings. *Vis. Res.* **117**, 100–104 (2015).
13. Lappin, J. S., Shelton, A. L. & Rieser, J. J. Environmental context influences visually perceived distance. *Percept. Psychophys.* **68**, 571–581 (2006).
14. Bodenheimer, B. et al. Distance estimation in virtual and real environments using bisection. In *Proceedings of the 4th Symposium on Applied Perception in Graphics and Visualization* (eds. Wallraven, C. & Sundstedt, V.) 35–40 (2007).

15. Norman, J. F. et al. The visual perception of long outdoor distances. *Sci. Rep.* **14**, 3207 (2024).
16. Da Silva, J. A. Scales for perceived egocentric distance in a large open field: comparison of three psychophysical methods. *Am. J. Psychol.* **98**, 119–144 (1985).
17. Da Silva, J. A., Ruiz, E. M. & Marques, S. L. Individual differences in magnitude estimates of inferred, remembered, and perceived geographical distance. *Bull. Psychon Soc.* **25**, 240–243 (1987).
18. Norman, J. F., Adkins, O. C., Norman, H. F., Cox, A. G. & Rogers, C. E. Aging and the visual perception of exocentric distance. *Vis. Res.* **109**, 52–58 (2015).
19. Norman, J. F., Dukes, J. M., Shapiro, H. K. & Peterson, A. E. The visual perception of large-scale distances outdoors. *Perception* **49**, 968–977 (2020).
20. Koenderink, J. J. Multiple visual worlds. *Perception* **30**, 1–7 (2001).

Author contributions

J.F.N. developed the study concept. J.F.N. and M.C. developed the study design. J.F.N., M.C., E.H., A.B.R., E.N.B., Y.M., and A.S. were responsible for stimulus preparation. Data collection was performed by J.F.N., M.C., E.H., A.B.R., E.N.B., Y.M., and A.S. The data analysis was performed by J.F.N. The figure preparation was performed by J.F.N. J.F.N. wrote the manuscript. All authors reviewed the manuscript.

Declarations

Competing interests

The authors declare no competing interests.

Additional information

Correspondence and requests for materials should be addressed to J.F.N.

Reprints and permissions information is available at www.nature.com/reprints.

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.